S282 Astronomy

Stellar populations – Part 1

Stellar motion in the solar neighbourhood

The Open University

Study time: 4 hours

Summary

In this extended spreadsheet activity you are supplied with data tables of observations of the radial velocities of several hundred stars within about 300 pc of the Sun. The activity is split into two parts. In this part of the activity you will analyse these data in order to measure the motions of stars in our region of the Galaxy. In the second part, another activity called *Population II stars in the solar neighbourhood*, you will consider evidence for different stellar populations.

You should have read to the end of Section 1.2.4 of *An Introduction to Galaxies and Cosmology* before starting this activity and you should understand the terms: Doppler effect, proper motion, Galactic coordinates, metallicity, Population I, and Population II. This activity builds on some of the ideas that were introduced in the activity *Stellar distance and motion*, and you may find it useful to review your work on that activity prior to starting this one, along with revising Section 3.2.1 of *An Introduction to the Sun and Stars*.

Learning outcomes

- Appreciate the characteristics of the various stellar populations in the Galaxy.
- Experience using and interpreting observational data (positions and velocities).
- Appreciate the dynamic state of the Galaxy and the motions of stars.
- Gain skills in using computational and analytical tools (in this instance, spreadsheets) to interpret astronomical data.

Introduction to the activity

During your study of the Milky Way you will have read about a number of characteristics of the Galaxy, such as the major structural components and their masses, and the sorts of objects associated with these components and the ways in which they are distributed through space. However, learning this collection of facts is only a small part of studying astronomy, or for that matter, science in general. An important part of understanding science involves understanding *how* the knowledge has been acquired. This is not a journey into history, but a journey into the scientific process.

Our current understanding of the Galaxy is built upon two inseparable pillars: observation and theory. Astronomical observations tell astronomers what the Galaxy looks like, but are influenced by the steps taken in making the observations. These steps may not provide a representative view of the Galaxy if factors influencing the observations have not been fully taken into account. Such a factor is absorption of light by dust, which is discussed in *An Introduction to Galaxies and Cosmology*. Astronomical theory, on the other hand, is built on a range of sciences including physics, mathematics, chemistry, geology/Earth sciences and biology that can be well studied on Earth. Often, however, Earth-based theory gives only a simplified treatment of complex astronomical phenomena, or does not deal with the same energy or size ranges found in astronomical settings. It is for these reasons that advances in astronomy require progress jointly in both theory and observations.

Observations are used to test and develop theory, and theory is used to help scrutinize and interpret observations. An important link between the two involves comparing observational data sets with models of the objects and/or phenomena studied. This often involves computing an idealized model of the data and comparing that with the real observations. This activity will take you through such a comparison, using a spreadsheet to handle the data manipulation.

Introduction to the science

You know already that most stars in the disc of the Milky Way move in orbits which are approximately circular. However, the motions are not perfectly circular, and there are slight differences in the motions of stars relative to one another. From our perspective as observers on the Earth, we can in theory measure two components of motion for each star:

1. the component along our line of sight to the star, which is called the radial velocity, and
2. the component across our line of sight to the star, which is its motion in the plane of the sky, called the transverse velocity.

Question 1

How might you measure the radial velocity of a star? (*Hint*: consider what you have learnt about the spectra of stars.)

What about the transverse velocity of a star; how would you measure that? Measuring a transverse velocity accurately is hard, because you need to calculate that velocity from two other measurements, both of which are difficult to make. The transverse velocity must be calculated from the measured distance of a star and its measured proper motion. Distances are often accurate to no better than a few per cent, and often they are a *lot* less accurate. The motions of stars across the sky, even over periods of a decade or more, are usually small compared with the apparent size of a stellar image, which is typically 1 arc second (usually abbreviated arcsec), so proper motions are also hard to measure accurately. This means that both the distance and proper motion of a star are often poorly known, and hence the star's transverse velocity is also poorly known. In contrast, it is quite straightforward to make measurements of the Doppler shift of a star. It can be performed to almost arbitrary precision, relatively easily to $\pm 1\ \mathrm{km\ s^{-1}}$, and to even finer precision, $\sim 5\ \mathrm{m\ s^{-1}}$, if extra effort is made.

How can an astronomer make reliable measurements of objects' velocities in the Galaxy, some at large distances from the Sun, if only one of the two components can be measured accurately? This may sound like a rhetorical question, but it isn't really. The answer is: by being clever.

The famous astronomer S. Chandrasekhar suggested in 1942 that the difficulty of measuring transverse velocities could be circumvented by measuring the radial velocities of stars in special directions. For example, if you observed stars in the direction on the sky directly opposite the Galactic centre, a direction that we call the Galactic anti-centre, the radial velocities of stars would correspond to their motions directly towards or away from the Galactic centre. The same applies to stars observed towards the Galactic centre.

From *An Introduction to Galaxies and Cosmology*, Section 1.2.4, you should recall that the Sun and other stars in the solar neighbourhood are orbiting the Galaxy clockwise when viewed from the north Galactic pole, moving more or less towards Galactic coordinates $l = 90°$ and $b = 0°$. If you measured the radial velocities of stars in this particular direction, then you would be measuring how much their orbital motion differs from that of the Sun. You could do the same by looking towards $l = 270°$ and $b = 0°$. Similarly, if you measure the radial velocities of stars by looking towards the north or south Galactic poles, then the radial velocities will reveal how fast stars are moving into and out of the Galactic disc in the vertical, or z, direction.

This technique allows astronomers to make accurate velocity measurements in these very important directions in the Galaxy, which are called 'cardinal directions'. The velocity components measured relative to the Sun in the three directions $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$ are given the symbols U, V, and W respectively, and are positive for motions directed away from the Sun towards those directions.

Question 2

Why don't we need to specify the value of l for motion in the direction corresponding to the W velocity?

Question 3

Sketch a diagram of the Galaxy showing the location of the Galactic centre and the Sun, and the three cardinal directions described above. Label them as the Galactic anti-centre, the rotation direction and the north Galactic pole (NGP). Also label these according to the velocity components, showing the directions corresponding to positive velocities.

Why are these directions important? For instance, why not make measurements at $l = 30°$ and $b = 40°$, say? Well, the cardinal directions are important because they probe the symmetry axes of the Galaxy. The motion measured in the direction of the Galactic anti-centre indicates the motion in or out of the Galaxy's central gravitational potential; motion in the rotation direction tells you about the rotational speed and hence the angular momentum of the stars; motion towards the Galactic poles tells you how much energy the stars have to rise temporarily above the plane of the Galaxy.

A series of observations exploiting this technique was made by astronomers A. Sandage and G. Fouts. They measured the radial velocities of approximately 1300 stars in three groups located within 10° of the Galactic anti-centre, rotation

direction, and NGP. They published the radial velocities they measured in *The Astronomical Journal*, March 1987, Volume 93, page 592–609. We will use their data to examine the motions of stars in the solar neighbourhood.

Question 4

Why do you think Sandage and Fouts measured stars within 10° of the cardinal directions, rather than exactly in the cardinal directions?

The activity

Data tabulations

Three data files (u.dat, v.dat, and w.dat) have been used in this activity, containing observations made by Sandage and Fouts. The data files are publicly available, and were obtained via the website

ftp://cdsarc.u-strasbg.fr/pub/cats/III/145/

You can download the files yourself if you wish, as they are quite small, but we have already provided the data from these files in spreadsheets on the S282 DVD to save time (details follow in the next sections of this activity).

The first five lines of the file u.dat are shown in Figure 1. The first column gives the name of the star, e.g. SAO76948. Columns 2 and 3 give the coordinates of the star in equatorial coordinates, i.e. right ascension (RA) and declination (Dec). Right ascension can be measured in degrees from 0° to 360°, but it is more usual to divide the 360° into 24 hours, each hour into 60 minutes, and each minute in 60 seconds, and then to express the right ascension in h, min and s. That is the convention that has been adopted in u.dat, and SAO76948 has a right ascension of 05h 01min 47s. The declination is measured in degrees, arc minutes, and arc seconds, so the declination of SAO76948 is 29° 54′ 27″. Column 4 indicates the spectral type of the star, columns 5 and 6 give its brightness as *V* and *B* magnitudes, and the final column gives its radial velocity in km s^{-1}.

SAO76948	50147	295427	K0	7.8	9.2	-40.4
SAO76958	50302	294156	F0	8.0	8.5	19.3
SAO76983	50608	293200	G5	8.9	9.8	6.4
SAO76988	50632	290453	G0	8.6	9.7	-20.5
SAO76989	50633	294411	F8	6.6		6.0

Figure 1 The first five lines of the file u.dat.

The spreadsheets

The three data files have already been loaded into a spreadsheet called
`s282_gc11_start.sxc`

(the Excel version of the file is called `s282_gc11_start.xls`).

- Start the S282 Multimedia guide program and open the folder called 'Our Galaxy', then click on the icon for this activity (Stellar populations – Part 1).
- Press the Start button to access the folder on the DVD containing the StarOffice and Excel versions of the raw data file.
- Open the file you wish to use by double-clicking on it.
- As soon as it is open, save it with a new name to your hard disk, using the File | Save As... option. Make sure you note which folder of your hard disk you save it into, so you can easily find it again.

Making histograms of the radial velocities

The first task is to examine the range of radial velocities which have been measured for the stars. In the spreadsheet file, you will find three sheets named U, V and W, which correspond to the three input files. To select a different sheet, you click on the named tab at the bottom of the screen.

- Begin by clicking on the tab for sheet W.

You can examine the velocities by using the Page Down and Page Up keys to scan through the table, but this will give you only a very cursory view of what is in the table. A more scientific approach is to plot a histogram of the velocities. A histogram is a graph which has the desired independent variable (in our case the W velocity) along the horizontal axis, arranged into intervals, also called bins, of some chosen width. The vertical axis plots the number of data entries that fall within each bin. An example of a histogram common to students would be one giving the number of students who have achieved certain grades in an examination. The histogram would have exam scores along the horizontal axis, perhaps in bins of 10 marks, and the number of students achieving this score along the vertical axis. An example of such a diagram is given in Figure 2.

Student	Mark out of 100	Min	Max	mid-range	Frequency
Anne	58	0	10	5	0
Bob	59	11	20	15	0
Cath	42	21	30	25	0
David	86	31	40	35	1
Erica	72	41	50	45	2
Felix	51	51	60	55	3
Gemma	82	61	70	65	2
Harry	39	71	80	75	3
Isabella	68	81	90	85	2
Joanne	75	91	100	95	0
Kevin	72		>100		0
Margaret	48				
Neil	63				

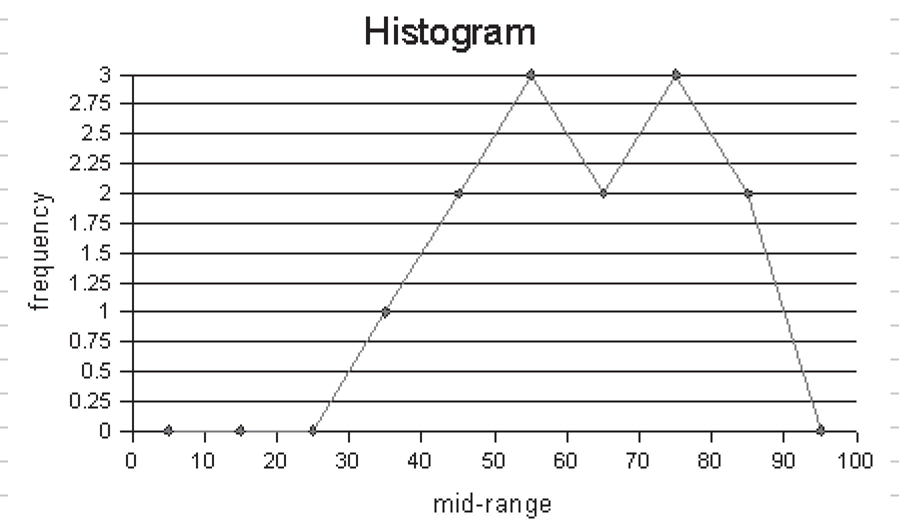

Figure 2 Example of a histogram and associated data tabulations showing the number of students scoring certain grades in an examination. The horizontal axis has been divided into bins 10 marks wide. (Histograms are usually, but not always, drawn as bar-graphs which have flat tops within each bin. Here, for convenience of plotting, we show them as single points at the mid values of each bin.)

In order to see what range of radial velocities is present in data file W, we should plot a histogram of the velocities in the same way we have for the examination scores in Figure 2. This is done as follows. (Note that a completed version of the spreadsheet (`s282_gc11_finish.sxc`) is provided on the DVD in the spreadsheets folder. If you have difficulties with your calculations, you may find it helpful to refer to the completed version to see how it has been done.)

First you need to discover what range of values will show up in the histogram. You could do this by scanning your eye through the file, but you can find out exactly by sorting the file according to the radial velocities, and then seeing what are the largest and smallest values.

- Click on any cell in the sheet and then type Ctrl-A to select (highlight) the entire table.

- Then sort the table according to the contents of column G, which is the radial velocity column. This is done by clicking on Data | Sort... | Sort by Column G | Ascending | OK. Note down the lowest and highest values in the radial velocity column (you will have to use Page Down to reach the bottom of the table), in this case –87.1 and 95.1 km s^{-1}.
- Finally, undo the sort so that you return to the original format of the file (Edit | Undo).

Next you need to decide how large each velocity bin should be in the histogram. Sandage and Fouts claim that each of their radial velocity measurements is accurate to about 4.7 km s^{-1}, so there is little to be gained by using velocity bins smaller than this. It helps to use nice round numbers, so intervals of 5 km s^{-1} would be appropriate, provided this gives more than just a few stars per velocity bin. If this produced too few stars per bin, we could increase the bin size to, say, 10 km s^{-1}.

Considering the data ranges found above, we will begin by using a velocity scale ranging from –100 to +100 km s^{-1}, binned over intervals of 5 km s^{-1}. To set up the histogram, we need to work out how many stars fall into each velocity bin. This count of stars per bin is called the 'frequency', and is easy to calculate with a spreadsheet. We then set out this information as a table from which we can generate plots.

- Construct a table in sheet W which indicates the lower and upper radial velocity (v_r) limit on each bin, and the midpoint velocity of each bin. It will be enough initially to enter these values for just the first two bins, those running from –100 km $s^{-1} \leq v_r <$ –95 km s^{-1}, and –95 km $s^{-1} \leq v_r <$ –90 km s^{-1}. You could begin by putting the heading 'lower limit' in cell M1 of sheet W, and then setting out the adjacent cells as in Figure 3.

M	N	O	P
lower limit	upper limit	midpoint	frequency
-100	-95	-97.5	
-95	-90	-92.5	

Figure 3 Layout of cells when beginning to set out a frequency table.

Once this is done, you can easily extend the table to the full velocity range as follows.

- Select (highlight) the first two rows of numerical values that you have just entered. To do this, depress and hold down the left-hand mouse button on the cell containing the value –100, and drag the cursor to the cell containing the number –92.5, then release the mouse button. You will see that at the lower right corner of the selected cells there is a small black square.
- Click and hold down the left-hand mouse button on this square, and then drag it vertically downwards, continuing to hold down the left-hand mouse button. You will see that a small counter appears; this indicates the numerical value that the first column will contain.
- Continue to drag the cursor down until this counter reaches the value 95, which is the lower limit of the last cell you need (the cell 95 km $s^{-1} \leq v_r <$ 100 km s^{-1}). Then release the mouse button. You will see that this operation has extrapolated the bin ranges over the full range required.

The next step is to obtain the frequency values, i.e. the number of stars which fall into each radial velocity bin. This is done using a built-in function in the spreadsheet called ‘frequency’. ‘Frequency’ calculates the number of data values that fall below the upper limit of each bin, but which equal or exceed the lower limit of the bin. It also calculates the number of data values that exceed the highest bin. (If you have set up the bins to cover the full data range then this number will be zero, but the spreadsheet computes it all the same.)

- Note down the first and last cells containing radial velocity values in sheet W; these should be cells G2 and G421.
- Note down the first and last cells that contain the upper boundaries of the velocity bins; these should be cells N2 and N41.
- Select the empty cells into which the frequency values should be put. In the column that we labelled frequency above, you should depress the left-hand mouse button in the cell just below the heading, and drag the cursor down to the right of the last entry in the midpoint column, 97.5, *and to the next cell down as well*, to allow the spreadsheet room to calculate the frequency of data values that exceed your last velocity bin. This process will cause you to highlight cells P2 to P42.
- Finally, with the cells still highlighted, begin to type in the formula to activate the frequency calculation. As this formula needs to know about two types of data, namely the individual radial velocities and the bin boundaries, it is a special kind of formula called a ‘matrix’ formula (‘array’ formula in Excel), and is entered slightly differently to formulae you have used before. With cells P2 to P42 highlighted, type the formula `=frequency(g2:g421;n2:n41)` but instead of just ending this with a normal carriage return (Enter), *use the three-key sequence* Ctrl-Shift-Enter (i.e. press Ctrl and hold it down, then press Shift and hold it down, and finally, whilst these two keys are held down, press Enter). You will then see that the highlighted cells will be filled with the number of stars whose radial velocities fall into each bin range.

It is now a simple exercise to plot the histogram of velocities.

- Select (highlight) the midpoint values for the bins, then hold down the Ctrl key and select (highlight) the frequency values, ignoring the final frequency cell (which should be a zero in any case) for which there is no corresponding midpoint value. *Check*: you should have highlighted the block of cells from O1 to P41.
- Clicking on Insert | Chart... will bring up the plot-building window, which takes you through four stages of constructing the plot. The second stage allows you to select the type of plot, where you should choose an XY chart, which is second from the left on the second line of choices. Also set Data series in Columns (if this is not already selected). The third stage allows you to Choose a variant; select Lines with symbols, which is the second option from the left. In completing the histogram, it is advisable to select Grid lines for both the X axis and Y axis. Give sensible axis labels as well, and a graph title. Once you have clicked on Create, the final graph will appear near the top of the sheet.
- You can move and/or resize the plot by clicking on its borders, and then dragging the boundaries with the cursor. Moving its upper-left corner to R6, and resizing its lower-right corner to Z27, is a convenient location and size.

- If you have a printer, you can print the graph and the associated frequency table. Highlight the block of cells from M1 to Z42, and then click on File | Print... . When the print dialogue window appears, chose Selection in the Print range.

You now have a histogram of the *W* velocity distribution. So, what does it mean? You will begin to interpret the data in the next section of this activity, but, just in case you are looking for a suitable point at which to take a break, this may be a good point at which to do so.

Interpreting the data

Now consider the histogram that you have plotted for the *W* velocities. It shows that the vast majority of stars have *W* velocities in the range –40 to +20 km s^{-1}. This suggests that the velocities that carry stars vertically above or below the disc are generally only a few tens of km s^{-1}. You might also note quite quickly that the distribution of velocities is not centred on zero, but is displaced to slightly negative velocities, centred around –10 km s^{-1}. Note that by definition, radial velocities are positive for stars that are receding and negative for stars that are approaching the observer. What does the non-zero average motion mean? It shows that stars viewed towards the North Galactic Pole are, on average, approaching the Sun. Why should this be the case? What is it about the Sun, or the position of the Sun in the Galaxy, which would make stars on average approach it? Consider this for a few minutes before proceeding.

In considering the question posed above, you might have proposed that the gravity of the Galactic disc is attracting more stars towards the disc than it allows to escape. If we were discussing gas clouds, you might indeed see this happening, but for stars there is a problem with such a proposition: stars almost never collide with anything substantial, so as they approach the disc with a certain speed, nothing absorbs their kinetic energy, and consequently they leave the disc on the opposite side with the same kinetic energy, and hence the same speed, that they approached it. That is, stars are not piling up near the centre of the disc. (In fact, the opposite occurs. Stars do interact with giant molecular clouds, but this tends to increase rather than decrease the heights to which stars travel, and hence causes a small net drift of stars away from the Galactic disc.)

The answer to the question, 'What is it about the Sun, or the position of the Sun in the Galaxy, which would make stars on average approach it?' is 'Nothing!' But if there is nothing special about the Sun, why do stars appear to be flowing towards it? The answer is that it is the Sun that is drifting relative to the neighbouring system of stars, rather than those stars drifting relative to us. The average motion of disc stars in the solar neighbourhood defines what is called the 'local standard of rest'.

You can make a precise measurement of the average *W* velocity of the sample observed towards the NGP above by calculating the average of their radial velocity values in the spreadsheet.

- If you click into an empty cell S1, and type the formula `=average(g2:g421)` then cell S1 will report the mean radial velocity, in this case –10.21 km s^{-1}.

When you perform a calculation of this sort, it is good practice to also insert a label saying what it is that you have calculated, and to give the appropriate physical units.

- Click into cell R1, and give the label average rv.
- Click into cell T1, and give the units km s^-1.

The negative sign on the radial velocity means that the mean radial velocity is directed towards the Sun. The finding that the mean speed of stars viewed in the direction of the NGP is about 10 km s^{-1} towards the Sun indicates that the Sun is rising up out of the disc, toward the NGP, at a speed of 10 km s^{-1}.

How accurately do we know this mean velocity? Obviously, adding one extra star to the sample would change the average slightly, so the average velocity depends on exactly which stars are included in our sample. In principle, we could measure so many stars that the effect of adding additional individual stars makes essentially no change to the average, but in reality it isn't feasible to do that. Consequently, we have to base our measurement of the mean velocity on just a sample of stars, in this case a few hundred of them. Fortunately, when the histogram of data values has a bell-shape, as the *W* velocity histogram does, there is a simple formula that gives an estimate of the uncertainty in the average of the data. The formula is used to calculate a quantity called 'the standard error in the mean', where the word 'mean' means 'average'. Sometimes this expression is abbreviated to just 'standard error'. The standard error in the mean depends on how spread out the data are and how many points have been measured. The spread of the data is measured by a quantity called the 'sample standard deviation', σ, which the spreadsheet will calculate. The standard error in the mean, *se*, is given by the sample standard deviation of the distribution divided by the square root of the number of points, *n*, in the sample:

$$se = \frac{\sigma}{\sqrt{n}}$$

To calculate the standard error in the mean, we therefore need to calculate the standard deviation and work out how many stars we have *W* velocities for.

- In cell S2, just below the calculation of the average, the sample standard deviation can be calculated by entering the formula `=stdev(g2:g421)`.
- In cell S3, the number of points can be calculated by using the formula `=count(g2:g421)`.
- In cell S4, the standard error can be calculated using the formula `=S2/SQRT(S3)`.

Cell S4 shows that the standard error in the mean *W* velocity is 1.06 km s^{-1}, so we conclude that the mean radial velocity of stars in the direction of the NGP is -10.2 ± 1.1 km s^{-1}.

What is the significance of this calculation for the *W* velocities? You have just measured the mean speed of the Sun in one of the three cardinal directions in the Galaxy, and have made an estimate of the uncertainty in that measurement.

In the next part of the activity, you will apply what you have learnt to the data measured in the two other cardinal directions to derive information on the *U* and *V* velocities as well. Before doing this, you should add appropriate labels and units to columns *R* and *T* respectively for the quantities you have just calculated. Now would be a good time to look back over what you have done so far in this activity, to clarify any uncertainties, and, if you wish, to take another break.

The three-dimensional space motion of the Sun

You now know enough about analysing the observational data in the spreadsheet to calculate the mean velocities for the other two cardinal directions as well, and hence to work out the full three-dimensional motion of the Sun. Return to the spreadsheet now, and calculate the mean motions for stars in the direction to the

Galactic anti-centre and the rotation direction. Although you do not need to view the histograms in order to calculate mean velocities, you will need to examine them later, so do plot them. When you have calculated the mean radial velocities and the uncertainties for the stars in each of the three cardinal directions, record the results in Table 1. When you have done so, answer the following question.

Table 1 Results of velocity measurements of stars in the three cardinal directions.

	U	*V*	*W*
Mean velocity/km s^{-1}			−10.2
Standard error in the mean/km s^{-1}			1.1

Question 5

What velocity components do you infer for the *motion of the Sun* in the Galaxy based solely on the mean radial velocities of stars from the observations by Sandage and Fouts? Be careful to explain in what direction the Sun is moving according to these calculations.

The total speed, υ_{LSR}, of the Sun relative to the local standard of rest can be calculated as

$$\upsilon_{\mathrm{LSR}} = \sqrt{u^2 + \upsilon^2 + w^2}$$

where u, υ and w are the velocities of the Sun relative to the LSR calculated in Question 5. Doing this for the Sun's measured motion, (−3, 13, 10) km s^{-1}, indicates that its speed relative to the LSR is 17 km s^{-1}.

It is possible to use the inferred velocity of the Sun to calculate its orbit in the Galaxy. Such a calculation is beyond the scope of this activity, but shows that the Sun travels no more than about 100 pc from the midplane of the Galactic disc. That is, it will remain within the disc in the future, so it is a member of the disc population, Pop. I. It is not a Pop. II star that just happens to be passing through the disc.

Be sure to save all of your results from this activity, as you will need them for Part 2 this stellar populations activity, *Population II stars in the solar neighbourhood*, and then attempt the final two questions.

Question 6

What other evidence is there (from your studies, rather than from this activity) that the Sun is a Pop. I star?

Question7

To complete your work on this activity, write a brief summary of the observations you have used, and what you have shown about the stars in the solar neighbourhood.

Answers to questions

Question 1

The radial velocities of stars can be measured via the Doppler shift of their spectra, since the Doppler shift indicates motion along the line of sight to an object.

Question 2

Motion in the W direction is towards the north Galactic pole, and since all lines of longitude converge at the pole, it is enough to specify just its Galactic latitude, which is $b = 90°$.

Question 3

Your figure should resemble Figure 4 below.

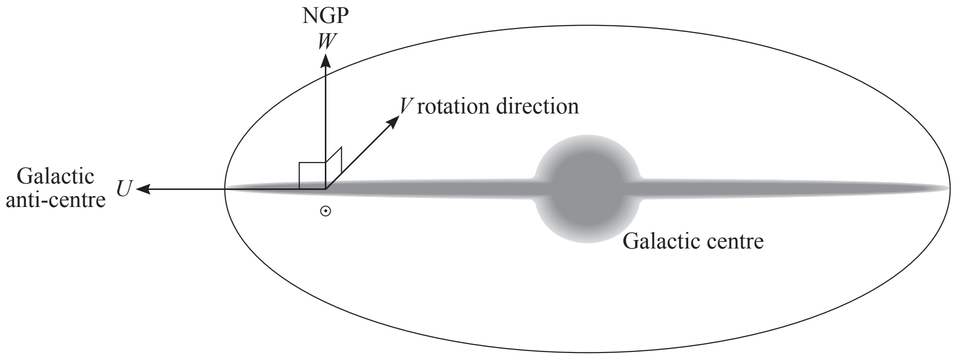

Figure 4 Schematic diagram of the Galaxy showing the cardinal directions corresponding to the velocity components U, V and W.

Question 4

Strictly, the cardinal directions are of infinitesimal size, so the probability of there being even one star exactly in any of these directions is negligible. This makes it necessary to measure stars within some angle of the cardinal directions. There is a compromise to be reached between going to a large enough angular separation to include a sufficiently large number of stars, yet not so far that the radial velocities no longer correspond closely enough to the desired velocity component U, V or W. Sandage and Fouts regarded the error made in going up to 10° away from the poles to be tolerable.

Question 5

As a result of your analysis you should have obtained values consistent with those given in Table 2.

Table 2 Results of velocity measurements in the three cardinal directions.

	U	V	W
Mean velocity/km s^{-1}	3.4	−13.2	−10.2
Standard error/km s^{-1}	1.5	1.4	1.1

The table shows that the uncertainties are in the range 1.1 to 1.5 km s^{-1}, so it is sensible to state the mean velocities to the nearest 1 km s^{-1}. (The mean value of some variable x is sometimes denoted $< x >$.) From the mean radial velocities of the observed stars, $(< U >, < V >, < W >) = (3, -13, -10)$ km s^{-1}, we infer that the Sun's (u, v, w) velocities relative to the local standard of rest are $(-3, 13, 10)$ km s^{-1}. That is, the Sun is moving towards the Galactic centre at 3 km s^{-1}, it is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}. These values compare very favourably with the generally accepted values, based on stars better representing the local standard of rest, $(-9, 12, 7)$ km s^{-1}.

Question 6

The Sun has a high metallicity, $Z \sim 0.02$, and its age is 4.5×10^9 yr, which are (respectively) higher and younger than are typical of Pop. II stars.

Question 7

We began with radial velocities for 420–450 stars in each of the three cardinal directions $(l, b) = (180°, 0°)$, $(90°, 0°)$ and $b = 90°$, located within about 300 pc of the Sun. By plotting velocity histograms, we found that most of these stars have velocities which differ from that of the Sun by no more than a few tens of km s^{-1}. By assuming that there is nothing special about the Sun or its location in the Galaxy, we have used the observed mean motions of the stars in the solar neighbourhood to infer the motion of the Sun relative to the local standard of rest. We find that the Sun is moving towards the Galactic centre at 3 km s^{-1}, is overtaking the motion of the local standard of rest by 13 km s^{-1} in the rotation direction, and moving towards the north Galactic pole at 10 km s^{-1}.